Quantum spherical spin glass with inverse power-law disorder

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We analyze the critical properties of a quantum spherical spin-glass model with random inverse power-law interactions $r^{-(d+\sigma)}$. It was shown in a previous publication that the effective partition function calculated with help of the replica method for the spin-glass fluctuating fields $Q_{\alpha\gamma}(\vec{k},\omega_1,\omega_2)$ separates into a mean-field contribution for the $Q_{\alpha\alpha}(0,\omega,-\omega)$ and a strictly long-range partition function for the fields $Q_{\alpha\neq\gamma}(\vec{k},\omega_1,\omega_2)$. Here $\alpha,\gamma=1,\ldots,n$ are replica indices. The long-range part W_{LR} describes a phase transition in a Q^3-field theory that we analyze using the renormalization group with dimensional regularization and minimal subtraction of dimensional poles. By generalizing standard field theory methods to our particular situation, we can identify the upper critical dimensionality as $d_c=\frac{5\sigma}{2}$ at very low temperatures due to the dimensionality shift $D_c=d_c+\frac{\sigma}{2}=3\sigma$. We then perform an $\epsilon'=d_c-d$ expansion to order one loop to calculate the critical exponents by solving the renormalization-group equations.

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I. INTRODUCTION

It has been known for some time that the critical behavior of classical systems with long-range (LR) inverse power-law interactions $V(r)=r^{-(d+\sigma)}$, where d is the space dimensionality, falls into different universality classes depending on the value of the range parameter $\sigma>0$. From the early results in the spherical model[1] to the renormalization-group calculations in φ^4-field theory[2] and φ^3-field theory,[3] the picture emerged that for $\sigma>\sigma_o$ the critical exponents take their short-range (SR) value, while for $0<\sigma<\sigma_o$ the system falls into a different universality class with LR σ-dependent exponents. The exact value of σ_o is still a controversial subject, as in Refs. 2 and 3 it is proposed that $\sigma_o=2$ with a discontinuity in the critical exponent η, while in Ref. 4 it is suggested that $\sigma_o=2-\eta_{\mathrm{SR}}$, where η_{SR} is the value of the exponent η calculated in the $\epsilon=4-d$ expansion for a φ^4-field theory with SR interactions. The problem is revisited in a more recent publication[5] where a numerical analysis is presented that is not completely conclusive. The properties of classical Ising spin glasses with long-range algebraic disorder were first studied in Ref. 6 and later in Ref. 7 with analogous conclusions that for $0<\sigma<2$ the critical behavior is of LR type, with σ-dependent critical exponents. In this case η_{SR} is negative and $\sigma_o=2$.

In the present paper we study the critical properties of the quantum spherical spin glass with LR algebraic disorder in the region $\sigma<2$ where the SR contributions are supposed to be irrelevant. We do not investigate here the crossover between SR and LR behaviors. A particular class of systems that presents a quantum critical point (QPC) is quantum spin glasses such as the insulating $LiHo_xY_{1-x}F_4$.[8] The Ising spin glass in a transverse field with random long-range interactions has been considered in Ref. 9. For general reviews on quantum phase transitions and quantum spin glasses we refer the reader to Refs. 10 and 11.

In a recent publication[12] we presented a detailed study of the critical properties of the quantum spherical spin glass with SR disorder by using renormalized perturbation theory with dimensional regularization.[13] From that work it emerged that there is a dimensional shift from the space dimensionality d to the effective $D=d+1$ due to the time dependence of the quantum operators and that scaling behavior in the critical region requires the introduction of a dynamical critical exponent z. The critical exponents are then calculated in an expansion in $\epsilon=6-D=5-d$.

We extend here the theory in Ref. 12 to the study of the critical properties of a quantum spherical spin-glass model with random inverse power-law interactions $r^{-(d+\sigma)}$. It has been pointed out that quantum fluctuations may drive the critical temperature to $T_c=0$ and that a new transition with a dimensional shift $D=d+1$ may occur at a QCP.[14] Although we showed that this conjecture is satisfied in the quantum spherical spin glass with short-range disorder,[12] we will show here that the dimensional shift satisfies a more general relation and becomes also σ dependent in the case of LR disorder. The space dimensionality plays an important role in phase transitions. There exists a critical dimension d_c, called the upper critical dimension, such that for $d>d_c$ the leading infrared behavior will be given by the free theory, while for $d<d_c$ the theory develops singularities that give rise to a critical behavior different from mean-field theory. There is also another critical dimension d_l, called the lower critical dimension, below which long-range order disappears. In some cases one can also make expansions of critical exponents for $d>d_l$, like in the nonlinear σ model where $d_l=2$.[13] While renormalization-group theory gives us a precise value of the upper critical dimension d_c, the evaluation of the lower critical dimension is not an easy task. In Ref. 15 a detailed analysis of d_l for classical vector spin glasses with inverse power-law disorder was presented, but we do not see how to extend this calculation to the quantum case.

Detailed discussions and lengthy formal proofs of the basic theory were presented in Ref. 12; hence, we present here only the mathematical expressions needed to follow our work, referring the interested reader to that paper for details. We obtained that the effective partition function calculated with help of the replica method for the spin-glass fluctuating fields $Q_{\alpha\gamma}(\vec{k},\omega_1,\omega_2)$ separates into a mean-field contribution for $Q_{\alpha\alpha}(0,\omega,-\omega)$ and a strictly long-range partition function for the fields $Q_{\alpha\neq\gamma}(\vec{k},\omega_1,\omega_2)$. Here $\alpha,\gamma=1,\ldots,n$ are replica indices. The mean-field part W_{MF} coincides with previous results obtained in the quantum spherical spin glass with infinite range interactions.[16] The long-range part W_{LR} describes a phase transition in a Q^3-field theory that we analyze using the renormalization group with dimensional regularization and minimal subtraction of dimensional poles. By generalizing standard field theory methods to this particular situation, we observe that scale invariance requires imaginary time and an inverse temperature to scale as $\beta=\Lambda^{\sigma/2}$, for Λ an inverse length, inducing a dimensional shift $D=d+\frac{\sigma}{2}$ at very low temperature. As the critical effective upper dimension is $D_c=3\sigma$, we obtain $d_c=\frac{5\sigma}{2}$ for fixed σ and we then perform an $\epsilon'=d_c-d$ expansion on the order of one loop to calculate the critical exponents by solving the renormalization-group equations.

The plan of the paper is as follows. We present in Sec. II the model and results, while we reserve Sec. III for discussions and comparison with previous work.

II. MODEL AND RESULTS

We consider a spin glass of quantum rotors with moment of inertia I in the spherical limit[12,16,17] with Hamiltonian

$$\mathcal{H}_{\mathrm{SG}}+\mu\sum_i S_i^2=\frac{1}{2I}\sum_i P_i^2-\frac{1}{2}\sum_{i,j}J_{ij}S_iS_j+\mu\sum_i S_i^2, \quad (1)$$

where the spin variables at each site are continuous $-\infty<S_i<\infty$ and we considered the canonical momentum P_i with commutation rules as follows:

$$[S_j,P_k]=i\delta_{j,k}. \quad (2)$$

The sum in Eq. (1) runs over sites $i,j=1,\ldots,N$. The bond coupling J_{ij} in Eq. (1) is an independent random variable with the Gaussian distribution[6,7]

$$P(J_{ij})=e^{-J_{ij}^2/2J^2V_{ij}}\sqrt{\frac{1}{2\pi J^2 V_{ij}}} \quad (3)$$

and $V_{ij}=|\vec{R}_i-\vec{R}_j|^{-(d+\sigma)}$ is a long-range site-dependent variance with Fourier transform at low momentum k,

$$V(k)\approx 1-k^{\sigma}. \quad (4)$$

The chemical potential μ is a Lagrange multiplier that insures the mean spherical condition

$$-\frac{\partial\langle\ln\mathcal{W}\rangle}{\partial(\mu)}=\sum_i\int_0^\beta d\tau\langle S_i^2\rangle=\beta N \quad (5)$$

and $\beta=1/T$ is the inverse temperature. We work in units where the Boltzmann constant $k_B=\hbar=1$ and $\mathcal{W}$ is the quantum partition function

$$\mathcal{W}=\mathrm{Tr}\,\exp\left[-\beta\left(\mathcal{H}_{\mathrm{SG}}+\mu\sum_i S_i^2\right)\right] \quad (6)$$

that can be expressed as a functional integral[12,18,19]

$$\mathcal{W}=\int\prod_i\mathcal{D}S_i\exp(-\mathcal{A}_{\mathcal{O}}-\mathcal{A}_{\mathcal{SG}}), \quad (7)$$

where the noninteracting action $\mathcal{A}_{\mathcal{O}}$ is given by

$$\mathcal{A}_{\mathcal{O}}=\int_0^\beta d\tau\sum_i\left[\frac{I}{2}\left(\frac{\partial S_i}{\partial\tau}\right)^2+\mu S_i^2(\tau)\right] \quad (8)$$

and the interacting part is

$$\mathcal{A}_{\mathcal{SG}}=\frac{1}{2}\sum_{i,j}J_{ij}\int_0^\beta d\tau\,S_i(\tau)S_j(\tau). \quad (9)$$

The free energy may be calculated with the replica method as

$$F=-\frac{1}{\beta N}\lim_{n\to 0}\frac{W_n-1}{n}, \quad (10)$$

where $\langle\mathcal{W}^n\rangle_{\mathrm{ca}}=W_n$ is the partition functional for n-identical replicas, configurationally averaged over the probability distribution of J_{ij} in Eq. (3). Following the method in Ref. 12 we obtain that W_n may be expressed as a functional over fluctuating spin-glass fields $Q_{\alpha\gamma}(\vec{k},\omega,\omega')$, where $\omega=\frac{2\pi m}{\beta}$ is a discrete Matsubara frequency for finite temperature and $\alpha,\gamma=1,\ldots,n$ are replica indices, which separates into two parts:

$$W_n=W_{\mathrm{MF}}W_{\mathrm{LR}}. \quad (11)$$

Here, W_{MF} is the mean-field functional for the fields $Q_{\alpha\alpha}(0,\omega,-\omega)$ already obtained in Ref. 16 that determines the chemical potential $\mu(T,I)$ through the spherical constraint in Eq. (5), while W_{LR} depends on the spin-glass fluctuations $Q_{\alpha\neq\gamma}(\vec{k},\omega,\omega')$ for long-range interactions and describes the critical behavior. We remark that these fields depend naturally on two independent times (frequencies) because the disorder is not time correlated and configurational average restores translational invariance in space, but not in the time direction. We obtain from Ref. 12

$$W_{\mathrm{LR}}=\int\prod_{\alpha\neq\gamma}\mathcal{D}Q_{\alpha\gamma}(\vec{k},\omega,\omega')\exp(-A_{\mathrm{LR}}\{Q\}), \quad (12)$$

where $\alpha,\gamma=1,\ldots,n$ are replica indices and

$$\begin{aligned}A_{\mathrm{LR}}\{Q\}=&\sum_{\alpha\neq\gamma}\sum_{\omega_1\omega_2}\int d\vec{k}\left[\frac{\mu-\mu_c}{\mu_c}+k^{\sigma}+s^2(\omega_1^2+\omega_2^2)\right]\\&\times Q_{\alpha\gamma}(\vec{k},\omega_1,\omega_2)Q_{\alpha\gamma}(-\vec{k},-\omega_1,-\omega_2)\\&+\frac{\lambda}{3!}\sum_{\alpha\neq\gamma\neq\delta}\sum_{\omega_1\omega_2\omega_3}\int d\vec{k}_1 d\vec{k}_2\,Q_{\alpha\gamma}(\vec{k}_1,\omega_1,\omega_2)\\&\times Q_{\gamma\delta}(\vec{k}_2,-\omega_2,\omega_3)Q_{\delta\alpha}(-\vec{k}_1-\vec{k}_2,-\omega_3,-\omega_1). \quad (13)\end{aligned}$$

The critical value $\mu_c=J$ in Eq. (13) determines the critical line $T_c(I)$ calculated in Ref. 16. Having in mind a renormalization-group calculation, the frequency term in the noninteracting inverse propagator is affected by the coefficient s^2, as it will turn out that momentum and frequency renormalize differently and their coefficients cannot be kept both equal to unity. The infinite volume limit was taken in

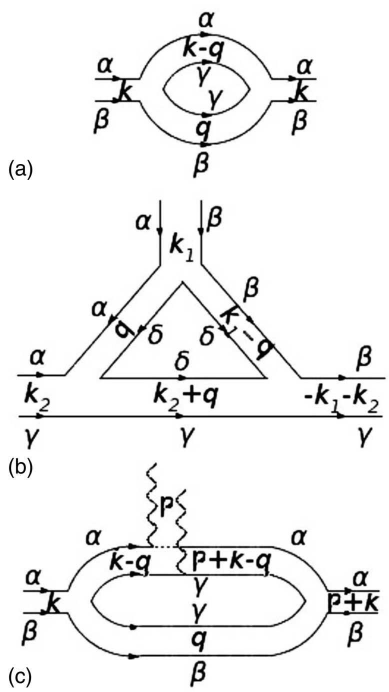

FIG. 1. Diagrammatic representation of the vertex functions. A double line represents a propagator with two replica indices α and γ, momentum $\vec{k}$, and two frequencies ω_1 and ω_2. Top: $\Gamma^{(2)}$, middle: $\Gamma^{(3)}$, and bottom: $\Gamma^{(2,1)}$.

Eq. (13), but for the moment the temperature is kept finite and the sums are over discrete Matsubara frequencies. In what follows it is implicit that $Q_{\alpha\gamma}$ stands for $Q_{\alpha\neq\gamma}$, while $Q_{\alpha}(\omega)$ stands for $Q_{\alpha\alpha}(0,\omega,-\omega)$. There is also a coupling $Q_{\alpha\alpha}(\vec{q}\neq 0)Q_{\alpha\neq\gamma}Q_{\gamma\neq\alpha}$ that presents a smaller degree of infrared divergence at the critical theory, when $\mu=\mu_c$, and can be neglected.[12]

We now proceed with the renormalization-group calculation using dimensional regularization and minimal subtraction of dimensional poles[13] to one loop order. In Eq. (13) we kept only the terms $O(Q^3)$ because the terms $O(Q^4)$ would be irrelevant close to the critical dimensionality of Q^3 theory, as there is no change in the sign of λ for the Gaussian probability distribution of the random bonds. To analyze the value of the critical dimensionality we consider separately the case of finite temperature from that of $T=0$. In both cases the vertex functions that present divergencies needing renormalization are the inverse propagator $\Gamma^{(2)}$, the three-point vertex function $\Gamma^{(3)}$, and the two-point vertex function with one insertion $\Gamma^{(2,1)}$.[13] To one loop order they are given by the diagrams in Fig. 1. At this point it is important to distinguish between the system temperature T and the critical parameter $t=\frac{\mu-\mu_c}{\mu_c}$ that measures the approach to criticality.

We start by analyzing the transition at finite temperature T. Since the action in Eq. (13) must be dimensionless, dimensional analysis tells us that for Λ an inverse length

$$[k]=\Lambda, \quad [Q]=\Lambda^{-(d+\sigma)/2}, \quad [\lambda]=\Lambda^{(3\sigma-d)/2}, \tag{14}$$

and the upper critical dimension is $d_c=3\sigma$, as corresponds to a classical system. The vertex functions calculated with the usual rules in φ^3-field theory[3,13] are

$$\Gamma^{(2)}(\vec{k},\omega_1,\omega_2)=\Gamma^{(0)}(\vec{k},\omega_1,\omega_2)-(n-2)\frac{1}{2}\lambda^2 \times\sum_{\omega}\int d\vec{p}\; G_0(\vec{p},\omega,\omega_1)G_0(\vec{k}-\vec{p},\omega_2,-\omega), \tag{15}$$

where

$$\Gamma^{(0)}(\vec{k},\omega_1,\omega_2)=t+k^{\sigma}+s^2(\omega_1^2+\omega_2^2)=G_0^{-1}(\vec{k},\omega_1,\omega_2) \tag{16}$$

and

$$\Gamma^{(3)}(\vec{k}_1,\vec{k}_2,\omega_1,\omega_2,\omega_3) =\lambda+(n-3)\lambda^3\sum_{\omega}\int d\vec{p}\; G_0(\vec{p},\omega_1,\omega)G_0(\vec{k}_1+\vec{p},-\omega,\omega_2) \times G_0(\vec{k}_1+\vec{k}_2+\vec{p},-\omega,\omega_3). \tag{17}$$

The theory will be renormalized at the critical point $t=0$. To get away from the critical point we should consider a perturbation expansion in t by means of the insertion[13]

$$\Delta A=\frac{1}{2!}\sum_{\gamma,\nu}\sum_{\omega_1\omega_2}\int d\vec{q}\; t(\vec{q})\int d\vec{p} \times Q_{\gamma\nu}(\vec{p},\omega_1,\omega_2)Q_{\nu\gamma}(\vec{q}-\vec{p},-\omega_2,-\omega_1) \tag{18}$$

that leads to a third singular vertex function $\Gamma^{(2,1)}$ with two external legs and one insertion shown in Fig. 1;

$$\Gamma^{(2,1)}(\vec{k},\vec{q},\omega_1,\omega_2)=1+(n-2)\lambda^2\sum_{\omega}\int d\vec{p}\; G_0(\vec{p},\omega,\omega_1) \times G_0(\vec{q}-\vec{p},-\omega_1,-\omega)G_0(\vec{k}+\vec{p},\omega,\omega_2). \tag{19}$$

At finite temperature T and critical $t=0$, the sums over Matsubara frequencies in the vertex functions have only one singular term with $\omega_i=0$, then we recover the transition for classical spin glasses described by an expansion in $\epsilon=3\sigma-d$.[7]

A different scenario emerges when T is near zero. For sufficiently low T the frequency sums may be replaced with integrals

$$\sum_{\omega}\rightarrow\beta\int_{-\infty}^{\infty}d\omega \tag{20}$$

and now all the frequencies contribute to the renormalization process. A look at $\Gamma^{(0)}$ in Eq. (16) tells us that the scale of frequencies and inverse temperature must now be

$$[\omega]=\Lambda^{\sigma/2}, \quad [\beta]=\Lambda^{-\sigma/2}, \quad [s]=\Lambda^{0}. \tag{21}$$

The vertex functions in Eqs. (15), (17), and (19) will be singular at an effective dimension $D_c=d_c+\frac{\sigma}{2}=3\sigma$, with the upper critical space dimension becoming $d_c=\frac{5\sigma}{2}$.

The calculation of the integrals in the method of dimensional regularization merits discussion. In the low-temperature limit the sum over frequencies are replaced with integrals as indicated in Eq. (20), and then we need for $\Gamma^{(2)}$ in Eq. (15), at the critical value $t=0$,[13]

$$I_2 = \int d\omega d\vec{p} \frac{1}{p^\sigma + s^2(\omega_1^2+\omega^2)} \frac{1}{[\vec{p}-\vec{k}]^\sigma + s^2(\omega^2+\omega_2^2)}. \tag{22}$$

Differently from the classical case[3] this integral cannot be performed in closed form due to the presence of the ω_i^2 terms in the denominator. To circumvent the above problem we refer to our knowledge of the renormalization-group calculations in classical φ^4- and φ^3-field theories,[3,4] where there are no terms proportional to k^σ in the perturbation expansion. The only corrections are proportional to k^2 and they are irrelevant in the LR region, while they exhibit dimensional poles in the SR region $\sigma=2$. This is also the case here and we renormalize the theory at the symmetry point[13] with external momenta $\vec{k}_i=0$ and frequencies $\omega_i=\kappa^{\sigma/2}$, where κ is the scale parameter.

We present unified results for the critical properties in an expansion in $\epsilon'=\frac{5\sigma}{2}-d$ to one loop order, in the LR case when $\sigma<2$ and in the SR case when $\sigma=2$ that were calculated before.[12] As frequencies renormalize differently than momenta the exponent z differs from unity, depending also on the range parameter σ and the dimensionality through the ϵ' expansion. The integrals over momentum and frequency of Eqs. (15), (17), and (19) are calculated at a space dimensionality d when they converge[3,13] and the singularities appear as dimensional poles in ϵ'. We obtain for the singular parts, to leading order in the coupling constant and $n=0$,

$$\Gamma^{(2)}_{\alpha\gamma}(\vec{k},\omega_1,\omega_2) = k^\sigma + s^2(\omega_1^2+\omega_2^2) - \frac{1}{\epsilon'}u_0^2\left[s(\omega_1^2+\omega_2^2) - \zeta\frac{1}{3s}k^2\right], \tag{23}$$

$$\Gamma^{(3)}_{\alpha\gamma\delta} = u_0\kappa^{\epsilon'/2}\left[1 - 3u_0^2\frac{1}{s\epsilon'}\right], \tag{24}$$

$$\Gamma^{(2,1)}_{\alpha\gamma} = 1 - 2u_0^2\frac{1}{s\epsilon'}, \tag{25}$$

where we considered the bare dimensionless coupling u_0 through

$$\frac{\sqrt{\pi}}{2}\Gamma\left(\frac{d}{2}\right)S_d = u_0^2\kappa^{\epsilon'} \tag{26}$$

and S_d is the surface of the unit sphere in d dimensions. The parameter ζ in Eq. (23) takes the value $\zeta(\mathrm{LR})=0$ when $\sigma<2$ and $\zeta(\mathrm{SR})=1$ when $\sigma=2$. In order to cancel the dimensional poles we must consider a renormalized dimensionless coupling u and renormalized vertex functions by means of renormalization of the field and insertion $Q^2_{\alpha\gamma}$ through the functions Z_Q and Z_{Q^2}. Similar to the classical problem, there is no need of field renormalization for the LR disorder, when $\zeta(\mathrm{LR})=0$ and $Z_Q=1$, giving $\eta=2-\sigma$. The correction to the frequency term ω_i^2 in $\Gamma^{(2)}$ in Eq. (21) requires renormalization of the frequency coefficient $s(u)$. All together we obtain

$$\Gamma^{(2)}_R(u) = Z_Q(u)\Gamma^{(2)}(u_0,s),$$

$$\Gamma^{(3)}_R(u) = [Z_Q(u)]^{3/2}\Gamma^{(3)}(u_0,s),$$

$$\Gamma^{(2,1)}_R(u) = \bar{Z}_{Q^2}(u)\Gamma^{(2,1)}(u_0), \tag{27}$$

where

$$u_0 = u\left[1 + \frac{6-\zeta}{2\epsilon'}u^2\right], \tag{28a}$$

$$Z_Q = 1 + \frac{\zeta}{3\epsilon'}u^2, \tag{28b}$$

$$\bar{Z}_{Q^2}(u) = 1 + \frac{2}{\epsilon'}u^2, \tag{28c}$$

$$s^2 = 1 + \frac{3-\zeta}{3\epsilon'}u^2. \tag{28d}$$

From Eq. (28a) we calculate the β function

$$\beta(u) = \kappa\frac{\partial u}{\partial\kappa}\bigg|_\lambda = -\frac{\epsilon'}{2}u\left[1 - \frac{6-\zeta}{\epsilon'}u^2\right] \tag{29}$$

that vanishes at the trivial fixed points $u^*=0$, stable for $\epsilon'<0$, and $u^{*2}=\frac{1}{6-\zeta}\epsilon'$, stable for $\epsilon'>0$. The results in Eqs. (28) and (29) for $\zeta=1$ were obtained in our previous SR calculation.[12] To obtain the critical exponents we have to solve the renormalization-group equations[13] for the vertex function $\Gamma^{(2)}_R(\vec{k},s\omega_i,t,u,\kappa)$ near criticality, where $t=\frac{\mu-\mu_c}{\mu_c}$. Now we have to take into account also the dependence of s on κ through the coupling u and that there is no field renormalization, so calling $y_i=s\omega_i$, $i=1,2$, we obtain the renormalization-group equation at the fixed point $\beta(u^*)=0$ as follows:

$$\left[\kappa\frac{\partial}{\partial\kappa} + \gamma_s^*\sum_i y_i\frac{\partial}{\partial y_i} - \theta t\frac{\partial}{\partial t} - \zeta\eta_{\mathrm{SR}}\right]\Gamma^{(2)}_R(\vec{k},y_i,t,\kappa) = 0, \tag{30}$$

where

$$\theta = \left[\kappa\frac{\partial}{\partial\kappa}\ln\bar{Z}_{Q^2}\right]_{u=u^*},$$

$$\gamma_s^* = \left[\kappa\frac{\partial}{\partial\kappa}\ln s\right]_{u=u^*},$$

$$\eta_{\mathrm{SR}} = \left[\kappa\frac{\partial}{\partial\kappa}\ln Z_Q\right]_{u=u^*}. \tag{31}$$

The solution for $\Gamma^{(2)}_R$ in the SR case was discussed in Ref. 12; hence, we concentrate here on the solution for LR disorder with $\zeta=0$ that has the scaling form

$$\Gamma_R^{(2)}(\vec{k},y_i,t,\kappa) = \Phi[\vec{k};y_i t\kappa^{\theta-\gamma_s^*}], \tag{32}$$

where Φ is a function of the joint variable $y_i t\kappa^{\theta-\gamma_s^*}$ and dimensional analysis tells us that for ρ an inverse length,

$$\Gamma_R^{(2)}(\vec{k},y_i,t,\kappa) = \rho^\sigma \Gamma_R^{(2)}\left(\frac{\vec{k}}{\rho},\frac{y_i}{\rho^{\sigma/2}},\frac{t}{\rho^\sigma},\frac{\kappa}{\rho}\right) = \rho^\sigma \Phi\left[\frac{\vec{k}}{\rho};\frac{y_i t\kappa^{\theta-\gamma_s^*}}{\rho^{3(\sigma/2)+\theta-\gamma_s^*}}\right]. \tag{33}$$

If we choose[13]

$$\rho = \kappa\left(\frac{t}{\kappa^\sigma}\right)^{1/(\sigma+\theta)}, \tag{34}$$

we obtain

$$\Gamma_R^{(2)}(\vec{k},y_i,t,\kappa) = \kappa^\sigma\left(\frac{t}{\kappa^\sigma}\right)^{\sigma\nu}\Phi\left[\frac{\vec{k}}{\kappa}\left(\frac{t}{\kappa^\sigma}\right)^{-\nu};\frac{y_i}{\kappa^{\sigma/2}}\left(\frac{t}{\kappa^\sigma}\right)^{-\nu z}\right], \tag{35}$$

from which we identify the space correlation length exponent

$$\xi = \left(\frac{t}{\kappa^\sigma}\right)^{-\nu}, \quad \nu^{-1} = \sigma + \theta = 2 - \eta + \theta \tag{36}$$

and the time correlation length exponent

$$\xi_t = \left(\frac{t}{\kappa^\sigma}\right)^{-\nu z} = \xi^z, \quad z = 1 - \gamma_s^*\frac{2}{\sigma}. \tag{37}$$

From Eqs. (28), (31), (36), and (37) we obtain the results for the critical exponents, at the nontrivial fixed point,

$$\nu^{-1} = \sigma - \frac{\epsilon'}{3}, \quad \eta = 2 - \sigma - \zeta\frac{\epsilon'}{15},$$

$$z = 1 + \frac{\epsilon'}{6\sigma}\left[1 - \frac{\zeta}{5}\right] = 1 + \frac{\epsilon'}{6\sigma} + \frac{\zeta}{4}\eta_{\mathrm{SR}}, \tag{38}$$

where in the LR region when $\sigma<2$ we have $\epsilon'=\frac{5\sigma}{2}-d$ and $\zeta=0$, while in the SR region we have $\sigma=2$, $\epsilon'=5-d$, and $\zeta=1$. The correlation length exponent ν varies continuously from the LR to the SR values while the dynamical exponent z exhibits the discontinuity in the exponent η. Scaling theory gives for the static spin-glass susceptibility $\chi^{-1}=\Gamma_R^{(2)}(0,0,t)\approx t^\gamma$, with $\gamma=\nu(2-\eta)=\nu\sigma$, from Eq. (35).

III. CONCLUSIONS

In the present paper we analyze the critical properties of a quantum spherical spin-glass model with long-range random interactions. Since the model allows for exact detailed calculations, we showed before[12] how the effective partition function calculated with help of the replica method for the spin-glass fluctuating fields $Q_{\alpha\gamma}(\vec{k},\omega_1,\omega_2)$ separates into a mean-field contribution for the $Q_{\alpha\alpha}(0,\omega,-\omega)$ and a strictly short-range partition function for the fields $Q_{\alpha\neq\gamma}(\vec{k},\omega_1,\omega_2)$. Here $\alpha,\gamma=1,\ldots,n$ are replica indices. The mean-field part W_{MF} coincides with previous results[16] and it was discussed in detail in Ref. 12 where we showed that it determines the chemical potential $\mu(T,I)$ through the spherical constraint in Eq. (5). The long-range part W_{LR} describes a phase transition in a Q^3-field theory, where the fluctuating fields depend on a position variable $\vec{r}$ and two imaginary time variables τ_1 and τ_2. Scale invariance requires frequencies to scale as $\Lambda^{\sigma/2}$ for Λ an inverse length, then by generalizing the renormalization group with dimensional regularization and minimal subtraction of dimensional poles[13] to this particular situation we can identify the upper critical dimension as $d_c=\frac{5\sigma}{2}$, for fixed σ, at very low temperatures due to the dimensionality shift $D_c=d_c+\frac{\sigma}{2}=3\sigma$. We then perform an $\epsilon'=d_c-d$ expansion on the order of one loop to calculate the critical exponents by solving the renormalization-group equations, and they are listed in Eq. (38). For fixed dimensionality d we would have[6] $\epsilon'=3(\sigma-\sigma_c)$, where $\sigma_c=d/3$. We notice that the correlation length exponent ν goes continuously from the σ-dependent LR values when $\sigma<2$ to the SR value for $\sigma=2$, while the dynamical exponent z shows a discontinuous behavior when $\sigma=2$, as it does the exponent η.

The quantum Ising spin glass in a transverse field with LR correlated disorder was considered in Ref. 9, where the problem is mapped into the general Landau theory of quantum spin glasses of M-component rotors presented in Ref. 20. Based on general properties of symmetry and invariance, the authors present an effective functional for spin-glass Q fields, and at some points we make contact with their results. Our fields, as theirs, are bilocal in time, but our result for the effective functional is simpler and more tractable by standard field theory methods. We showed[12] that the partition functional separates exactly into a mean-field part for the replica diagonal $Q_{\alpha\alpha}(k=o,\omega,-\omega)$ and a long-range part for the fluctuating $Q_{\alpha\neq\beta}(k,\omega_1,\omega_2)$ in Eq. (11), while in the quantum spin glass with LR disorder considered in Refs. 9 and 20 the replica diagonal $Q_{\alpha\alpha}(\omega)$ is considered as an order parameter and a Landau functional is constructed for fluctuations diagonal in replica space around it. As a consequence of having different propagators, the renormalization-group equations, critical exponents, and the critical dimensionality obtained in Ref. 9 differ from ours. We conclude that, in the case of long-range disorder considered here, the quantum spherical spin-glass model belongs to a different universality class than the model in Ref. 9.

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